

Imperial College London Mathematics School Admissions Test Mark Scheme

Specimen Paper 2

Mark Scheme

Marking instructions

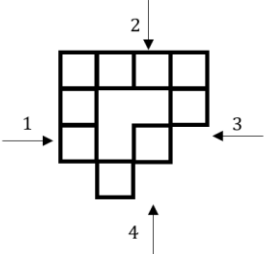
- Each question in sections A and B scores 2 marks for the correct answer or zero for no answer, the wrong answer or more than one answer.
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- For section C
 - M marks are for working and are given for a correct method, clearly shown even if there are some errors of arithmetic.
 - A marks are for the correct answer from correct working and can only be given if all the M marks so far in that part of the question have been earned.
 - B marks are independent marks.
 - Candidates may use any correct method; if this method is not in the mark scheme, award marks in a way that is as similar as possible to the methods shown in the mark scheme.

Section A

Number	Solution	Mark	Guidance
			Either 2 or zero for each question on Section A.
1	C A circle of radius 3 cm	2	Example reasoning Area of A is 24, B is 25, C is 9π which is a little over 27, D is 27 and E is 8π
2	D $3\frac{1}{3}$ square units	2	Example reasoning When $x = 0$, $y = -2$. When $y = 0$, $x = 10/3$ Area = $\frac{1}{2} \times 2 \times 10/3 = 10/3 = 3\frac{1}{3}$
3	A 2×10^7	2	Example reasoning This can be rewritten as $\frac{32 \times 10^5 \times 45 \times 10^3}{72 \times 10^2}$. This simplifies to 20×10^6 . = 2×10^7 in SI form
4	D 9:4	2	Example reasoning PQR is similar to SQT with length scale factor $\frac{2}{3}$. Let the height of PQR perpendicular PR to be h . The height of SQT will be $\frac{2}{3}h$ and $ST = \frac{2}{3}PR$. Area of PQR = $\frac{1}{2}PRh$, area of SQT = $\frac{1}{2} \cdot \frac{2}{3}PR \cdot \frac{2}{3}h = \frac{1}{2} \cdot \frac{4}{9}PRh$ ratio $1:\frac{4}{9}$ i.e. 9:4
5	C £20	2	Example reasoning The youngest receives $\frac{4}{5} \times \frac{5}{6} = \frac{2}{3}$ of the eldest's amount $\frac{2}{3}e + \frac{5}{6}e + e = 50$ multiply by 6 $4e + 5e + 6e = 300$ giving $e = 20$
6	D $5 \times \frac{p^3}{10q^2} \times \frac{3}{(pq)^3}$	2	Example reasoning Considering indices for q : A is $q^{4-3} = q$, B is $q^{3-2} = q$, C is q , D is $q^{-2-3} = q^{-5}$, E is $q^{-3+5-1} = q$

Number	Solution	Mark	Guidance
7	A Their sum must be a multiple of 3	2	<p>Example reasoning</p> <p>If the first number is n then the 3 consecutive numbers add up to $3n + 3$</p> <p>This is a multiple of 3 for all n.</p> <p>Counter examples can be found for statements B, C, D e.g. B any even n, C any odd n, D $n = 2$</p> <p>E cannot be true if A is true.</p>
8	A $\frac{1}{2}$	2	<p>Example reasoning</p> <p>$P(3) + P(6) = \frac{1}{3}$, $P(2) = \frac{1}{12}$, $P(5) = \frac{1}{12}$</p> <p>$P(2) + P(3) + P(5) + P(6) = \frac{1}{3} + \frac{1}{12} + \frac{1}{12} = \frac{1}{2}$</p> <p>So $P(1) + P(4) = 1 - \frac{1}{2} = \frac{1}{2}$</p>
9	B 4	2	<p>Example reasoning</p> <p>The only points satisfying $x > 0$, $y > 0$ and $x + y < 5$ are (1,1), (1,2), (1,3), (2,1), (2,2) and (3,1)</p> <p>From these, all but (1,2) and (1,3) satisfy $y < 2x$ so there are 4 points</p>
10	B $3\left(1 - \frac{\pi}{4}\right)$	2	<p>Example reasoning</p> <p>A quarter of the circle has area $\frac{\pi}{4}$.</p> <p>The small square, with side OQ, minus a quarter circle has area $1 - \frac{\pi}{4}$.</p> <p>There are two complete sections like this and two sections that are half of this making 3 sections in total so the total shaded area is $3\left(1 - \frac{\pi}{4}\right)$</p>

Section B

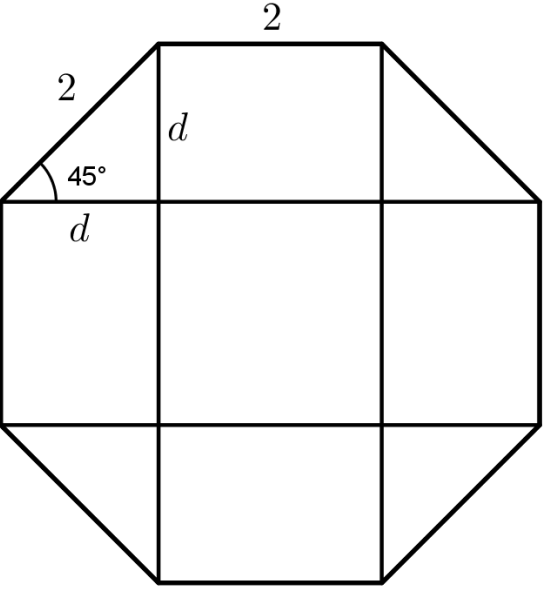
Number	Solution	Mark	Guidance
11	D 20	2	<p>Either 2 or zero for each question on Section B.</p> <p>Example reasoning</p> <div style="text-align: center;">  </div> <p>If the elevation is in direction 1 the maximum number of cubes is $4 \times 3 + 2 \times 2 + 2 \times 1 + 1 \times 2 = 20$</p> <p>From direction 2 it is $2 \times 3 + 2 \times 2 + 2 \times 1 + 3 \times 2 = 18$</p> <p>From direction 3 it is $1 \times 3 + 2 \times 2 + 2 \times 1 + 4 \times 2 = 17$</p> <p>From direction 4 it is $3 \times 3 + 2 \times 2 + 2 \times 1 + 2 \times 2 = 19$</p>
12	C 6	2	<p>Example reasoning</p> <p>For the numbers to still be prime when the digits are swapped, the tens digit must be 1, 3, 7 or 9. In the given range, the only primes that do this are 31 and 37.</p> <p>$37 - 31 = 6$</p>

Number	Solution	Mark	Guidance
13	E The country with the largest population won the most medals.	2	<p>Example reasoning</p> <p>A is not true as the smallest population (< 1 million) won 1 medal but the three larger countries from the table won 0 medals.</p> <p>B is not true as there are points showing smaller countries having more medals than some larger countries.</p> <p>C is not true as one country with just under 10 million population won 17 medals</p> <p>D cannot be a conclusion as relative wealth is not indicated</p> <p>E is true, the largest country with a population of 44 million won 20 medals (the most)</p>
14	E 176	2	<p>Example reasoning</p> <p>There are 11 numbers starting with 1 from 1 to 99 (1 and 10 – 19). Every number from 100 – 199 starts with 1 so there will be $11 + n - 99 = n - 88$ numbers starting with 1 from 1 to n (for the given values of n).</p> $\frac{n-88}{n} = \frac{1}{2} \text{ gives } n = 176$
15	C Around the same amount of rain fell in Beijing in May, June and July of 2015 as fell in August September and October of 2015.	2	<p>Example reasoning</p> <p>1987</p> <p>MJJ $\approx 2.1 + 3 + 3.5 = 8.6$</p> <p>ASO $\approx 8 + 1.1 + 0.5 = 9.6$</p> <p>Total $8.6 + 9.6 = 18.2$</p> <p>2015</p> <p>MJJ $\approx 1.1 + 1.6 + 3.7 = 6.4$</p> <p>ASO $\approx 2.8 + 3 + 0.6 = 6.4$</p> <p>Total $8.6 + 9.6 = 12.8$</p> <p>A no, $8.6 < 9.6$ B no, $18.2 \neq 2 \times 12.8$ C yes, $6.4 = 6.4$ (or close values) D no, $9.6 \neq 6.4$ E no, not wettest in 2015</p>

Number	Solution	Mark	Guidance
16	B 7	2	<p>Example reasoning</p> <p>There is 1 way that the total of the first two digits is 1 (10) and 2 ways for the total of the last two digits to be 1 (10 and 01) giving 1010 and 1001</p> <p>For a total of 2: 1 way for the first two (11) and 3 for the last two (20, 11 and 02) giving 1120, 1111, 1102.</p> <p>For a total of 3: 1 way for the first two (12) and 2 for the last two (12 and 21) giving 1212, 1221.</p> <p>Giving $2 + 3 + 2 = 7$</p>
17	A $2^2 \times 3 \times 5^2$	2	<p>Example reasoning</p> <p>6000 does not divide by 9 so eliminate B and D</p> <p>4500 does not divide by 8 so eliminate C</p> <p>The two remaining options have $2^2 \times 3$ as factors</p> <p>Dividing all three by $(2^2 \times 3) = 12$ gives 450, 375, 500</p> <p>All are divisible by 25 but 450 is not divisible by 125 so eliminate E.</p> <p>Alternatively</p> <p>$\text{hcf}(5\ 400, 4\ 500, 6\ 000) = 2^2 \times 3 \times 5^2$</p>
18	E 120	2	<p>Example reasoning</p> <p>The sequence is $(n + 1)^2 - 1$ (can cut up and reassemble to $n + 1$ sided squares with a 1×1 square missing)</p> <p>$(10 + 1)^2 - 1 = 121 - 1 = 120$</p>

Number	Solution	Mark	Guidance
19	E 100	2	<p>Example reasoning</p> <p>Let a be the number of starfish, b the number of shrimp and c the number of octopuses</p> $a + b + c = 200$ $4a + 6b + 8c = 1400$ $4a + 6b + 8c = 1400$ $\underline{6a + 6b + 6c = 1200}$ $-2a \quad + 2c = 200$ $c - a = 100$
20	B 25%	2	<p>Example reasoning</p> $\frac{1}{a} - \frac{1}{b} = \frac{3}{5} \left(\frac{1}{a} + \frac{1}{b} \right)$ $\frac{5}{a} - \frac{5}{b} = \frac{3}{a} + \frac{3}{b}$ $\frac{2}{a} = \frac{8}{b}$ $2b = 8a$ $a = \frac{1}{4}b$

Section C

Number	Solution	Mark	Guidance
21	 <p> $d = 2 \sin 45^\circ = \sqrt{2}$ (alternatively $d = 2 \cos 45^\circ$ or $2d^2 = 4$) </p> <p> Area of octagon = $4 \times \frac{1}{2} \times \sqrt{2} \times \sqrt{2} + 4 \times 2 \times \sqrt{2} + 2 \times 2$ $= 8 + 8\sqrt{2}$ </p> <p> Area of square = $2 \times 2 = 4$ </p> <p> Shaded area = $8 + 8\sqrt{2} - 4 = 4 + 8\sqrt{2} \text{ cm}^2$ </p>	<p>A1</p> <p>M1 A1</p> <p>A1 B1</p>	<p>Award if d is stated as $\sqrt{2}$ or the correct length is marked as $\sqrt{2}$ on a diagram. Allow $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$</p> <p>Award M1 for area of octagon using their values. A1 for $8 + 8\sqrt{2}$ or equivalent</p> <p>Allow equivalent using $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$ i.e. $4 + \frac{16}{\sqrt{2}}$</p> <p>Allow ft from incorrect area of octagon only. B1 for correct units.</p>

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	<p>Alternative method</p> $d = 2 \sin 45^\circ = \sqrt{2}$ <p>(alternatively $d = 2 \cos 45^\circ$ or $2d^2 = 4$)</p> $\text{Shaded area} = 4 \times (2 \times \sqrt{2}) + 4 \times \left(\frac{1}{2} \times \sqrt{2} \times \sqrt{2}\right)$ $= 4 + 8\sqrt{2} \text{ cm}^2$	<p>A1</p> <p>M1 A1</p> <p>A1 B1</p>	<p>Award if d is stated as $\sqrt{2}$ or the correct length is marked as $\sqrt{2}$ on a diagram. Allow $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$.</p> <p>M1 evidence of splitting shaded shape into rectangles and triangles. A1 if these are correct.</p> <p>Allow equivalent using $\frac{2}{\sqrt{2}}$ for $\sqrt{2}$ i.e. $4 + \frac{16}{\sqrt{2}}$</p> <p>Allow ft from incorrect area. B1 for correct units.</p>																																																
		[5]																																																	
22 a)	<p>If the red dice is a 2 then the other 2 must sum to 4, 7 or 10</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2">B + Y = 4</th> <th colspan="2">B + Y = 7</th> <th colspan="2">B + Y = 10</th> </tr> <tr> <th>B</th> <th>Y</th> <th>B</th> <th>Y</th> <th>B</th> <th>Y</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>3</td> <td>1</td> <td>6</td> <td>4</td> <td>6</td> </tr> <tr> <td>2</td> <td>2</td> <td>2</td> <td>5</td> <td>5</td> <td>5</td> </tr> <tr> <td>3</td> <td>1</td> <td>3</td> <td>4</td> <td>6</td> <td>4</td> </tr> <tr> <td></td> <td></td> <td>4</td> <td>3</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td>5</td> <td>2</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td>6</td> <td>1</td> <td></td> <td></td> </tr> </tbody> </table>	B + Y = 4		B + Y = 7		B + Y = 10		B	Y	B	Y	B	Y	1	3	1	6	4	6	2	2	2	5	5	5	3	1	3	4	6	4			4	3					5	2					6	1			<p>B3</p>	<p>Award 3 marks for all 12 correct possibilities listed with no incorrect possibilities</p> <p>Award 2 marks for 9 - 11 correct possibilities</p> <p>Award 1 mark for 5 - 8 correct possibilities</p>
B + Y = 4		B + Y = 7		B + Y = 10																																															
B	Y	B	Y	B	Y																																														
1	3	1	6	4	6																																														
2	2	2	5	5	5																																														
3	1	3	4	6	4																																														
		4	3																																																
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22 b)	<p>For $B + Y = 4$, $R + B + Y = 6$</p> <p>For $B + Y = 7$, $R + B + Y = 9$</p> <p>For $B + Y = 10$, $R + B + Y = 12$</p> <p>The only ones that give a square number are $B + Y = 7$</p> <table border="1" data-bbox="376 437 620 896"> <thead> <tr> <th colspan="2" data-bbox="376 437 620 496">$B + Y = 7$</th> </tr> <tr> <th data-bbox="376 496 497 555">B</th> <th data-bbox="497 496 620 555">Y</th> </tr> </thead> <tbody> <tr> <td data-bbox="376 555 497 614">1</td> <td data-bbox="497 555 620 614">6</td> </tr> <tr> <td data-bbox="376 614 497 673">2</td> <td data-bbox="497 614 620 673">5</td> </tr> <tr> <td data-bbox="376 673 497 732">3</td> <td data-bbox="497 673 620 732">4</td> </tr> <tr> <td data-bbox="376 732 497 791">4</td> <td data-bbox="497 732 620 791">3</td> </tr> <tr> <td data-bbox="376 791 497 850">5</td> <td data-bbox="497 791 620 850">2</td> </tr> <tr> <td data-bbox="376 850 497 896">6</td> <td data-bbox="497 850 620 896">1</td> </tr> </tbody> </table>	$B + Y = 7$		B	Y	1	6	2	5	3	4	4	3	5	2	6	1	B2	<p>Award 2 marks for all 6 correct possibilities listed with no incorrect possibilities</p> <p>Award 1 mark for 3 - 5 correct possibilities or ft selecting correct possibilities from their answer to (a)</p>
$B + Y = 7$																			
B	Y																		
1	6																		
2	5																		
3	4																		
4	3																		
5	2																		
6	1																		
		[5]																	

Number	Solution	Mark	Guidance
23 a)	$1 + 2 + 3 + \dots + 12 = 78$	A1	
23 b)	The sum of the 4 edges will use the numbers 10, 11, 12 and n twice and everything else once $(1 + 2 + 3 + \dots + 12) + 10 + 11 + 12 + n$ $= 78 + 33 + n$ $= 111 + n$	M1 A1	M1 for using the corner values twice A1 for correct simplification to $111 + n$
23 c)	The total along each edge is the same and must be an integer. There are 4 edges hence the overall sum is a multiple of 4.	B1	Any correct explanation that identifies that the total for each side can be found by dividing by 4.
23 d)	$\frac{111 + n}{4} = 27 + \frac{n + 3}{4}$ $n + 3$ is a multiple of 4 $n + 3 = 4$ gives $n = 1$ $n + 3 = 8$ gives $n = 5$ $n + 3 = 12$ gives $n = 9$ The next value would be greater than 12.	M1 A2	M1 for any correct strategy (using the equation shown or by trying values for n) A2 for identifying all 3 correct values for n A1 for identifying 2 of the correct values for n

<p>23 e)</p>	<p>For $n = 1$ the total along each edge is $27 + \frac{4}{4} = 28$ The available numbers to complete the square once the corners have been inserted are 2, 3, 4, 5, 6, 7, 8 and 9. The left hand column starts with a 12 and finishes with 11 so the other two numbers must add up to 5. This can only be done using the 2 and 3. The top row starts with a 12 and ends with a 10 so the other two numbers add up to 6. None of the remaining numbers will do this so $n = 1$ does not work.</p> <p>For $n = 5$ the total along each edge is $27 + \frac{8}{4} = 29$ The available numbers to complete the square once the corners have been inserted are 1, 2, 3, 4, 6, 7, 8 and 9. The left hand column needs 6 more to make this total. This can only be done using the 2 and the 4. The top row needs 7 more to make 29. This must use the 6 and the 1. The right hand column needs 14 more to make 28. No pair of numbers from the remaining digits (3, 7, 8 and 9) add up to 14 so $n = 5$ does not work.</p> <p>For $n = 9$ the total along each edge is $27 + \frac{12}{4} = 30$ The available numbers are 1, 2, 3, 4, 5, 6, 7 and 8. The left hand column needs 7 more to make 30. This can be done using the 6 and the 1. The top row needs 8 more to make 30. This can be done using the 5 and the 3. The right hand column needs 11 more to make 30. This can be done with the 7 and the 4. The bottom row needs 10 more to make 30. This can be done using the 8 and the 2. $n = 9$ is the only one that works.</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>Any correct explanation identifying that the 4 totals can not be made using the values available</p> <p>Any correct explanation identifying that the 4 totals can not be made using the values available</p> <p>Confirming correctly that $n = 9$ does give a possible solution.</p>
		<p>[10]</p>	

