Imperial College London Mathematics School Admissions Test

Mark Scheme

Marking instructions

- Each question in sections A and B scores 2 marks for the correct answer or zero for no answer, the wrong answer or more than one answer.
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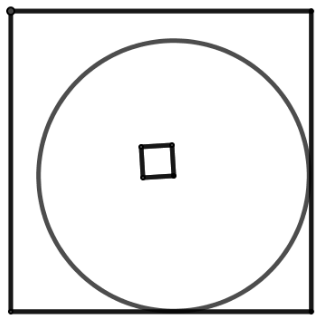
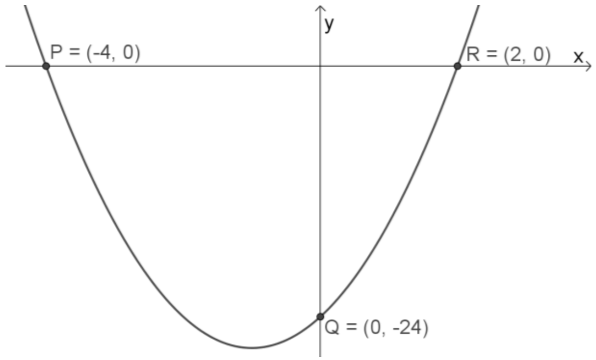
- For section C
  - M marks are for working and are given for a correct method, clearly shown even if there are some errors of arithmetic.
  - A marks are for the correct answer from correct working and can only be given if all the M marks so far in that part of the question have been earned.
  - B marks are independent marks.
  - Candidates may use any correct method; if this method is not in the mark scheme, award marks in a way that is as similar as possible to the methods shown in the mark scheme.

Section A

Number	Solution	Mark	Guidance
1	**A** They are both the same.	**2**	Either 2 or zero for each question on Section A. **Example reasoning** They can each be calculated by working out $\frac{p \times 25}{100}$
2	**C** $\frac{12ab^5}{8a^4} = 1.5a^{-4}b^5$	**2**	**Example reasoning** $\frac{12ab^5}{8a^4}$ is equal to $1.5a^{-3}b^5$ so this is the false one.
3	**D** 1:4	**2**	**Example reasoning** Area of rectangle is $\frac{1}{2}\mathrm{PQ} \cdot \mathrm{RQ}$ PT is $\frac{1}{2}\mathrm{PQ}$ so area of triangle is $\frac{1}{4}\mathrm{PQ} \cdot \mathrm{RQ}$; a quarter of the rectangle.
4	**D** 60	**2**	**Example reasoning** $660m + 8400n = 60(11m + 140n)$ 11 and 140 have no common factor. 60 is always a factor.
5	**B** There is exactly one prime number in the sequence.	**2**	**Example reasoning** $n^2 + 2n - 3 = (n+3)(n-1)$ so $n^2 + 2n - 3$ has two factors unless one of the brackets is 1. This can only be when $n - 1 = 1$ so when $n = 2$
6	**E** $A = \frac{2V}{r} + 2\pi r^2$	**2**	**Example reasoning** The total surface area consists of two circles and the curved surface area. The two circles together have area $2\pi r^2$. The curved surface area opens out to form a rectangle of length equal to the circumference of the circle and width equal to the height, h, of the cylinder. Curved surface area = $2\pi rh = \frac{2V}{r}$

Number	Solution	Mark	Guidance
7	**C** No, the mean has to be bigger than 22.	**2**	**Example reasoning** With 9 buses at the station, the total number of people is $9 \times 25 = 225$. There must be at least one person on the 10th bus to drive it. The smallest number of people on the 10 buses is 226. So the mean is 22.6 or more.
8	**E** b is greater than a for all values of x and there are some values of x for which a is greater than c.	**2**	**Example reasoning** $b = a + 5$ so $b > a$ for all values of x. Could a be greater than c? If $a > c$ then $3x + 5 > 5x + 10$. $-5 > 2x$ so $-2.5 > x$. There are values of x for which a is greater than c.
9	**D** 4	**2**	**Example reasoning** The centre of the circle can be anywhere in a square of side 2mm 

Number	Solution	Mark	Guidance
10	**A** (-1, -27)	**2**	**Example reasoning** The turning point must lie on the line of symmetry, so the *x*-coordinate is -1. The *y*-coordinate must be below -24. 

Section B

Number	Solution	Mark	Guidance
11	**C** There is a country in Europe where more than half the population are over 50.	**2**	Either 2 or zero for each question on Section B. **Example reasoning** A one mobile phone each would be 100 phones per 100 people and most countries have more than that so it's not average. B There is positive correlation but that does not mean that one factor causes the other. C The highest median age is about 56. This means that half the population are 56 are over so more than half the population are over 50.
12	**D** $1785 \div 870$	**2**	**Example reasoning** $\text{speed} = \frac{\text{distance}}{\text{time}}$ The greatest speed is when the largest possible distance is divided by the smallest possible time.
13	**D** 984	**2**	**Example reasoning** You can subtract multiples of 4 or 40 from 1000. 40 is a multiple of 4. You can add multiples of 4 to 1000. 850 is 150 below 1000. 150 is not a multiple of 4. 874 is 26 below 1000. 26 is not a multiple of 4. 930 is 70 below 1000. 70 is not a multiple of 4. 984 is 16 below 1000. 16 is a multiple of 4..

<table>
<tr><th>Number</th><th>Solution</th><th>Mark</th><th>Guidance</th></tr>
<tr><td>14</td><td>A 2010 to 2011</td><td>2</td><td>Example reasoning
<table>
<tr><th>Year</th><th>Price (p)</th><th>Change from previous year (p)</th></tr>
<tr><td>2010</td><td>111</td><td></td></tr>
<tr><td>2011</td><td>129</td><td>18</td></tr>
<tr><td>2012</td><td>141</td><td>12</td></tr>
<tr><td>2013</td><td>140</td><td>1</td></tr>
<tr><td>2014</td><td>138</td><td>2</td></tr>
<tr><td>2015</td><td>118</td><td>20</td></tr>
</table>
A and E are possible answers.
Percentage change 2010 to 2011 is
$\frac{18}{111}\times 100=\frac{6}{37}\times 100$
Percentage change 2014 to 2015 is
$\frac{20}{138}\times 100=\frac{10}{69}\times 100$
$\frac{6}{37}\approx\frac{6}{36}=\frac{1}{6}$; $\frac{10}{69}\approx\frac{10}{70}=\frac{1}{7}$; $\frac{1}{6}>\frac{1}{7}$</td></tr>
<tr><td>15</td><td>E Two of the three events are equally likely</td><td>2</td><td>Example reasoning
Heads and tails are equally likely.
Exactly two heads is the same as exactly one tail so exactly one head is as likely as exactly two heads.</td></tr>
</table>

<table>
<tr><th>Number</th><th>Solution</th><th>Mark</th><th>Guidance</th></tr>
<tr><td>16</td><td>B 3</td><td>2</td><td>Example reasoning
<table>
<tr><th>No. of jumpers</th><th>No. of balls wool</th><th>No. balls wool left</th></tr>
<tr><td>1</td><td>7</td><td>45</td></tr>
<tr><td>2</td><td>14</td><td>38</td></tr>
<tr><td>3</td><td>21</td><td>31</td></tr>
<tr><td>4</td><td>28</td><td>24</td></tr>
<tr><td>5</td><td>35</td><td>17</td></tr>
<tr><td>6</td><td>42</td><td>10</td></tr>
<tr><td>7</td><td>49</td><td>3</td></tr>
</table>
The number of balls of wool left from jumpers must be a multiple of 3 so that they are all used on scarves. The multiples of 3 in the last column are 45, 24 and 3.</td></tr>
<tr><td>17</td><td>A 1FREE</td><td>2</td><td>Example reasoning
<table>
<tr><th>Offer</th><th>How many are paid for?</th></tr>
<tr><td>1FREE</td><td>8</td></tr>
<tr><td>25OFF</td><td>9</td></tr>
<tr><td>HALF</td><td>9</td></tr>
</table></td></tr>
<tr><td>18</td><td>C
<table>
<tr><th>Min</th><th>Max</th></tr>
<tr><td>7</td><td>9</td></tr>
</table></td><td>2</td><td>Example reasoning
You can see 6 cubes in the second view so it can’t be done in less than 6. Making the second shape from 6 cubes in a set of steps would not give the first view so at least 7 cubes are needed.
The two diagrams below show the views from the top with the number of cubes vertically from the bottom of the shape for the min and max number of cubes.

</td></tr>
</table>

<table>
<tr><th>Number</th><th>Solution</th><th>Mark</th><th>Guidance</th></tr>
<tr><td>19</td><td>A 40%</td><td>2</td><td>Example reasoning
If p % of the mixture comes from powder X then $\left(100-p\right)$ % comes from powder Y.
$0.59p+0.89\left(100-p\right)=77$
$89-0.3p=77$
$12=0.3p$
$p=\frac{12}{0.3}=40$</td></tr>
<tr><td>20</td><td>B 3</td><td>2</td><td>Example reasoning
For a square with 4 numbers, the numbers are
<table>
<tr><td>n</td><td>$n+1$</td></tr>
<tr><td>$n+7$</td><td>$n+8$</td></tr>
</table>
The answer is $(n+1)(n+7)-n(n+8)=7$

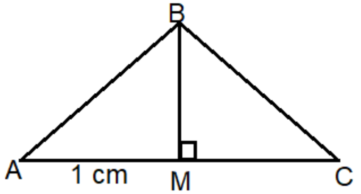
For a square with 9 numbers, the numbers are
<table>
<tr><td>n</td><td>$n+1$</td><td>$n+2$</td></tr>
<tr><td>$n+7$</td><td>$n+8$</td><td>$n+9$</td></tr>
<tr><td>$n+14$</td><td>$n+15$</td><td>$n+16$</td></tr>
</table>
The answer is $(n+2)(n+14)-n(n+16)=28$

Squares with 16 numbers all give the answer 63

It’s not possible to get a square with 25 numbers.</td></tr>
</table>

Section C

Number	Solution	Mark	Guidance
21	$\left(x+\frac{1}{x}\right)^2$	**M1**	Deciding to square
	$=x^2+\frac{1}{x^2}+2$	**A1**	Correct expression
	$x^2+\frac{1}{x^2}=3^2-2$	**M1**	
	7	**A1**	Correct answer from correct working
	Alternative method $x^2-3x+1=0$		
	$x=\frac{3\pm\sqrt{5}}{2}$	**M1**	
	$\left(\frac{3+\sqrt{5}}{2}\right)^2+\left(\frac{2}{3+\sqrt{5}}\right)^2=\frac{14+6\sqrt{5}}{4}+\frac{4}{14+6\sqrt{5}}$	**M1**	
	$\frac{7+3\sqrt{5}}{2}+\frac{2\left(7-3\sqrt{5}\right)}{49-45}=\frac{7+7}{2}=7$	**A1**	Getting 7 from correct working for one of the roots
	$\left(\frac{3-\sqrt{5}}{2}\right)^2+\left(\frac{2}{3-\sqrt{5}}\right)^2=\frac{14-6\sqrt{5}}{4}+\frac{4}{14-6\sqrt{5}}$		
	$\frac{7-3\sqrt{5}}{2}+\frac{2\left(7+3\sqrt{5}\right)}{49-45}=\frac{7+7}{2}=7$	**A1**	Showing that the other root gives the same answer Correct answer from correct working
		[4]	

Number	Solution	Mark	Guidance
22	Angle BAC Is exterior angle of polygon so 360°÷12	**M1**	Or finding interior angle of polygon (150°) OR PAB**C**= 120°
	PBAC= 30°	**A1**	OR PABM= 60° OR equivalent angle in other right angled triangle
		**M1**	Splitting the triangle into two right angled triangles
	$\cos 30° = \frac{1}{AB}$	**M1**	
	$\frac{\sqrt{3}}{2} = \frac{1}{AB}$	**M1**	Use of correct value for $\cos 30°$
	AB = $\frac{2}{\sqrt{3}}$ cm	**A1**	Correct answer from correct working
	Alternative method for last 4 marks $\frac{AB}{\sin 30°} = \frac{2}{\sin 120°}$	**M1**	
	AB = $2 \times \frac{1}{2} \div \frac{\sqrt{3}}{2}$	**M2**	M1 for each of: correct value for sin 30°, sin 120°
	AB = $\frac{2}{\sqrt{3}}$ cm	**A1**	Correct answer from correct working
	Another alternative method for last 4 marks Triangle ABM is half an equilateral triangle	**M1**	or $\text{BM} = \frac{\text{AB}}{2}$

	$AB^2 = 1 + \left(\frac{AB}{2}\right)^2$	**M1**	
	$\frac{3AB^2}{4} = 1$	**M1**	
	AB = $\frac{2}{\sqrt{3}}$ cm	**A1**	Correct answer from correct working

Number	Solution	Mark	Guidance
23 (a)	Circumference of circle is πD	**M1**	
	Perimeter of shaded region is $\frac{\pi d}{2}+\frac{\pi(D-d)}{2}+\frac{\pi D}{2}$	**M1**	Correct expressions for at least two semicircles
	$\frac{\pi d+\pi D-\pi d+\pi D}{2}=\pi D$	**A1**	Convincing completion to show that the perimeter of the shaded region is the same as the circumference of the original circle
	Alternative method Semicircle with diameter BC is $\frac{d}{D}$ of the semicircle with diameter AC	**M1**	One semicircle as a fraction of the length of the semicircle with diameter AC (or of the whole large circle)
	Semicircle with diameter AB is $\frac{(D-d)}{D}$ of the semicircle with diameter AC		
	Total fraction of the whole semicircle is $\frac{d}{D}+\frac{(D-d)}{D}$ =1	**M1**	Finding both semicircles above (or all three semicircles) as a fraction of the large semicircle (or whole large circle)
	So total perimeter of the shaded area is two semicircles with diameter AC or the same as the circumference of the circle with diameter AC	**A1**	Clear correct conclusion from correct working

Number	Solution	Mark	Guidance
23(b)	Area of semicircle with diameter BC is $\frac{\pi d^2}{8}$	**M1**	Allow correct and clear expressions using radius
	Area of semicircle with diameter AB is $\frac{\pi\left(D-d\right)^2}{8}$	**M1**	If wrong denominator is used for semicircles then just deduct one method mark as long as the denominators are consistent
	Area of circle with diameter AC is $\frac{\pi D^2}{4}$	**M1**	
	Area of shaded region is $\frac{\pi d^2}{8}+\frac{\pi D^2}{8}-\frac{\pi\left(D-d\right)^2}{8}$	**M1**	
	$\frac{\pi\left(d^2+D^2-D^2-d^2+2Dd\right)}{8}$	**M1**	
	Fraction of circle is $\frac{\pi Dd}{4}\div\frac{\pi D^2}{4}$	**M1**	
	$\frac{\pi Dd}{4}\times\frac{4}{\pi D^2}=\frac{d}{D}$	**A1**	Convincing completion from correct working
	Alternative method Semicircles are similar	**M1**	May be implied by later working rather than stated explicitly
	Area of semicircle with diameter BC is $\left(\frac{d}{D}\right)^2$ of the semicircle with diameter AC	**M1**	
	Area of semicircle with diameter AB is $\frac{\left(D-d\right)^2}{D^2}$ of the semicircle with diameter AC	**M1**	

Number	Solution	Mark	Guidance
	Shaded region as a fraction of semicircle with diameter AC is $1+\frac{d^2}{D^2}-\frac{(D-d)^2}{D^2}$	M1	
	$\frac{D^2+d^2-D^2-d^2+2Dd}{D^2}$	M1	
	$\frac{2d}{D}$ of the large semicircle	M1	
	So $\frac{d}{D}$ of the circle with diameter AC	A1	
		[7]	